Question	Scheme	Marks	AOs
1 (a)	$\lim_{\delta x \to 0} \sum_{x=2.1}^{6.3} \frac{2}{x}\delta x = \int_{2.1}^{6.3} \frac{2}{x}\,dx$	B1	1.2
		(1)	
(b)	$= [2\ln x]_{2.1}^{6.3} = 2\ln 6.3 - 2\ln 2.1$	M1	1.1b
	$= \ln 9$ CSO	A1	1.1b
		(2)	
			(3 marks)
Notes:			

Mark (a) and (b) as one

(a)

B1: States that $\int_{2.1}^{6.3} \frac{2}{x}\,dx$ or equivalent such as $2\int_{2.1}^{6.3} x^{-1}\,dx$ but must include the limits and the d*x*.

Condone $dx \leftrightarrow \delta x$ as it is very difficult to tell one from another sometimes

(b)

M1: Know that $\int \frac{1}{x}\,dx = \ln x$ and attempts to apply the limits (either way around)

Condone $\int \frac{2}{x}\,dx = p\ln x$ (including $p = 1$) or $\int \frac{2}{x}\,dx = p\ln qx$ as long as the limits are applied.

Also be aware that $\int \frac{2}{x}\,dx = \ln x^2$, $\int \frac{2}{x}\,dx = 2\ln|x| + c$ and $\int \frac{2}{x}\,dx = 2\ln cx$ o.e. are also correct

$[p\ln x]_{2.1}^{6.3} = p\ln 6.3 - p\ln 2.1$ is sufficient evidence to award this mark

A1: CSO ln 9 . Also answer = $\ln 3^2$ so $k = 9$ is fine. Condone $\ln|9|$

The method mark must have been awarded. Do not accept answers such as $\ln\frac{39.69}{4.41}$

Note that solutions appearing from "rounded" decimal work when taking lns should not score the final mark. It is a "show that" question

E.g. $[2\ln x]_{2.1}^{6.3} = 2\ln 6.3 - 2\ln 2.1 = 2.197 = \ln k \Rightarrow k = e^{2.197} = 8.998 = 9$

Question	Scheme	Marks	AOs
2	$\int x^3 \ln x \,dx = \frac{x^4}{4}\ln x - \int \frac{x^4}{4}\times\frac{1}{x}\,dx$	M1	1.1b
	$= \frac{x^4}{4}\ln x - \frac{x^4}{16}(+c)$	M1 A1	1.1b 1.1b
	$\int_1^{e^2} x^3 \ln x \,dx = \left[\frac{x^4}{4}\ln x - \frac{x^4}{16}\right]_1^{e^2} = \left(\frac{e^8}{4}\ln e^2 - \frac{e^8}{16}\right) - \left(-\frac{1^4}{16}\right)$	M1	2.1
	$= \frac{7}{16}e^8 + \frac{1}{16}$	A1	1.1b
		(5)	
			(5 marks)

Notes:

M1: Integrates by parts the right way round.

Look for $kx^4 \ln x - \int kx^4 \times \frac{1}{x}\,dx$ o.e. with $k > 0$. Condone a missing dx

M1: Uses a correct method to integrate an expression of the form $\int kx^4 \times \frac{1}{x}\,dx \to c\,x^4$

A1: $\int x^3 \ln x \,dx = \frac{x^4}{4}\ln x - \frac{x^4}{16}$ $(+\,c)$ which may be left unsimplified

M1: Attempts to substitute 1 and e^2 into an expression of the form $\pm px^4 \ln x \pm qx^4$, subtracts and uses $\ln e^2 = 2$ (which may be implied).

A1: $\frac{7}{16}e^8 + \frac{1}{16}$ o.e. Allow $0.4375e^8 + 0.0625$ or uncancelled fractions. NOT ISW: $7e^8 + 1$ is A0

....

You may see attempts where substitution has been attempted.

E.g. $u = \ln x \Rightarrow x = e^u$ and $\frac{dx}{du} = e^u$

M1: Attempts to integrate the correct way around condoning slips on the coefficients

$$\int x^3 \ln x \,dx = \int e^{4u} u \,du = \frac{e^{4u}}{4}u - \int \frac{e^{4u}}{4}\,du$$

M1 A1: $\int x^3 \ln x \,dx = \frac{e^{4u}}{4}u - \frac{e^{4u}}{16}(+c)$

M1 A1: Substitutes 0 and 2 into an expression of the form $\pm pue^{4u} \pm qe^{4u}$ and subtracts

....

It is possible to use integration by parts "the other way around"

To do this, candidates need to know or use $\int \ln x \,dx = x\ln x - x$

FYI $I = \int x^3 \ln x \,dx = x^3(x\ln x - x) - \int (x\ln x - x)\times 3x^2 \,dx = x^3(x\ln x - x) - 3I + \frac{3}{4}x^4$

Hence $4I = x^4 \ln x - \frac{1}{4}x^4 \Rightarrow I = \frac{1}{4}x^4 \ln x - \frac{1}{16}x^4$

Score M1 for a full attempt at line 1 (condoning bracketing and coefficient slips) followed by M 1 for line 2 where terms in I o.e. to form the answer.

Question	Scheme	Marks	AOs
3(a)	States or uses $h=1.5$	B1	1.1a
	Full attempt at the trapezium rule $=\frac{...}{2}\{1.63+2.63+2\times(2+2.26+2.46)\}$	M1	1.1b
	= awrt 13.3 or $\frac{531}{40}$	A1	1.1b
		(3)	
(b)(i)	$\int_3^9 \log_3(2x)^{10}\,dx = 10\times\text{"13.3"} = \text{awrt}133$ or e.g. $\frac{531}{4}$	B1ft	2.2a
(ii)	$\int_3^9 \log_3 18x\,dx = \int_3^9 \log_3(9\times 2x)\,dx = \int_3^9 2+\log_3 2x\,dx$ $=[2x]_3^9+\int_3^9 \log_3 2x\,dx = 18-6+\int_3^9 \log_3 2x\,dx = ...$	M1	3.1a
	Awrt 25.3 or $\frac{1011}{40}$	A1ft	1.1b
		(3)	
			(6 marks)
Notes:			

(a)

B1: States or uses $h=1.5$

M1: A full attempt at the trapezium rule.

Look for $\frac{\text{their } h}{2}\{1.63+2.63+2\times(2+2.26+2.46)\}$ but condone copying slips

Note that $\frac{\text{their } h}{2}1.63+2.63+2\times(2+2.26+2.46)$ scores M0 unless the missing brackets are recovered or implied by their answer. You may need to check.

Allow this mark if they add the areas of individual trapezia e.g.

$\frac{\text{their } h}{2}\{1.63+2\}+\frac{\text{their } h}{2}\{2+2.26\}+\frac{\text{their } h}{2}\{2.26+2.46\}+\frac{\text{their } h}{2}\{2.46+2.63\}$

Condone copying slips but must be a complete method using all the trapezia.

A1: awrt 13.3 (Note full accuracy is 13.275) or exact equivalent.

Note that the calculator answer is 13.324 so you must see correct working to award awrt 13.3

Use of $h=-1.5$ leading to a negative area can score B1M1A0 but allow full marks if then stated as positive.

(b)(i)

B1ft: Deduces that $\int_3^9 \log_3(2x)^{10}\,dx = 10\times\text{"13.3"} = \text{awrt } 133$

FT on their 13.3 look for 3sf accuracy but follow through on e.g. their rounded answer to part (a) so if 13 was their answer to part (a) then allow 130 here **following a correct method**.

A correct method must be seen here but a minimum is e.g. 10 × "13.3" = "133"

Note that $\int_3^9 \log_3(2x)^{10}\,dx = 133.2414316...$ so a correct method must be seen to award marks.

Attempts to apply the trapezium rule again in any way score M0 as the instruction in the question was to use the answer to part (a).

(b)(ii)

M1: Shows correct log work to relate the given question to part (a)

Must reach as far as e.g. $[2x]_3^9 + \int_3^9 \log_3 2x \, dx = ...$ with correct use of limits on $[2x]_3^9$ which may be implied or equivalent work e.g. finds the area of the rectangle as 2×6

A1ft: **Correct working** followed by awrt 25.3 but ft on their 13.3 so allow for 12 + their answer to part (a) **following correct work** as shown.

Note that $\int_3^9 \log_3 18x \, dx = 25.32414...$ **so a correct method must be seen to award marks**.

Some examples of an acceptable method are:

$$\int_3^9 \log_3 18x \, dx = \int_3^9 \log_3 (9 \times 2x) \, dx = \int_3^9 2 + \log_3 2x \, dx = 6 \times 2 + "13.3" = 25.3$$

$$\int_3^9 \log_3 18x \, dx = \int_3^9 \log_3 (9 \times 2x) \, dx = \int_3^9 2 + \log_3 2x \, dx = 12 + "13.3" = 25.3$$

$$\int_3^9 \log_3 18x \, dx = \int_3^9 \log_3 (9 \times 2x) \, dx = \int_3^9 2 + \log_3 2x \, dx = [2x]_3^9 + \int_3^9 \log_3 2x \, dx = 25.3$$

BUT just $12 + "13.3" = 25.3$ **scores M0**

Attempts to apply the trapezium rule again in any way score M0 as the instruction in the question was to use the answer to part (a).

Question	Scheme	Marks	AOs
4(a)	$(\text{f}'(x)=)4\cos\left(\frac{1}{2}x\right)-3$	M1 A1	1.1b 1.1b
	Sets $\text{f}'(x)=4\cos\left(\frac{1}{2}x\right)-3=0\Rightarrow x=$	dM1	3.1a
	$x=14.0$ Cao	A1	3.2a
		(4)	
(b)	Explains that $\text{f}(4)>0$, $\text{f}(5)<0$ **and** the function is continuous	B1	2.4
		(1)	
(c)	Attempts $x_1=5-\frac{8\sin 2.5-15+9}{"4\cos 2.5-3"}$ (NB f(5) = –1.212… and $\text{f}'(5)$ = –6.204…)	M1	1.1b
	x_1 = awrt 4.80	A1	1.1b
		(2)	
			(7 marks)
Notes:			

(a)

M1: Differentiates to obtain $k\cos\left(\frac{1}{2}x\right)\pm\alpha$ where α is a constant which may be zero and no other terms. The brackets are not required.

A1: Correct derivative $\text{f}'(x)=4\cos\left(\frac{1}{2}x\right)-3$. Allow unsimplified e.g. $\text{f}'(x)=\frac{1}{2}\times 8\cos\left(\frac{1}{2}x\right)-3x^0$

There is no need for $\text{f}'(x)=...$ or $\frac{\text{d}y}{\text{d}x}=...$ just look for the expression and the brackets are not required.

dM1: For the complete strategy of proceeding to a value for *x*.

Look for

- $\text{f}'(x)=a\cos\left(\frac{1}{2}x\right)+b=0,\quad a,b\neq 0$
- Correct method of finding a valid solution to $a\cos\left(\frac{1}{2}x\right)+b=0$

Allow for $a\cos\left(\frac{1}{2}x\right)+b=0\Rightarrow\cos\left(\frac{1}{2}x\right)=\pm k\Rightarrow x=2\cos^{-1}(\pm k)$ where $|k|<1$

If this working is not shown then you may need to check their value(s).

For example $4\cos\left(\frac{1}{2}x\right)-3=0\Rightarrow x=1.4...$ or 11.1… (or 82.8… or 637…. or 803 in degrees) would indicate this method.

A1: Selects the correct turning point $x=14.0$ and not just 14 or unrounded e.g. 14.011…
Must be this value only and no other values unless they are clearly rejected or 14.0 clearly selected. Ignore any attempts to find the *y* coordinate.
Correct answer with no working scores no marks.

(b)

B1: See scheme. Must be a full reason, (e.g. change of sign and continuous)
Accept equivalent statements for $\text{f}(4)>0$, $\text{f}(5)<0$ e.g. $\text{f}(4)\times\text{f}(5)<0$,"there is a change of sign", "one negative one positive". A minimum is "change of sign and continuous" but do **not** allow this mark if the comment about continuity is clearly incorrect e.g. "because *x* is continuous" or "because the interval is continuous"

(c)

M1: Attempts $x_1 = 5 - \frac{f(5)}{f'(5)}$ to obtain a value following through on their $f'(x)$ as long as it is a "changed" function.

Must be a correct N-R formula used – may need to check their values.

Allow if attempted in degrees. For reference in degrees f (5) = –5.65… and f ′(5) = 0.996… and gives $x_1 = 10.67...$

There must be clear evidence that $5 - \frac{f(5)}{f'(5)}$ is being attempted.

so e.g. $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \Rightarrow x_1 = 4.80$ scores M0 as does e.g. $x_1 = x - \frac{8\sin\left(\frac{1}{2}x\right) - 3x + 9}{4\cos\left(\frac{1}{2}x\right) - 3} = 4.80$

BUT evidence may be provided by the accuracy of their answer. Note that the full N-R accuracy is 4.804624337 so e.g. 4.805 or 4.804 (truncated) with no evidence of incorrect work may imply the method.

A1: $x_1 =$ awrt 4.80 not awrt 4.8 but isw if awrt 4.80 is seen. Ignore any subsequent iterations.

Note that work for part (a) cannot be recovered in part (c)

Note also:

$5 - \frac{f(5)}{f'(5)} =$ awrt 4.80 following a correct derivative scores M1A1

$5 - \frac{f(5)}{f'(5)} \neq$ awrt 4.80 with no evidence that $5 - \frac{f(5)}{f'(5)}$ was attempted scores M0

Question	Scheme	Marks	AOs
5(a)	e.g. $\frac{3kx-18}{(x+4)(x-2)} \equiv \frac{A}{x+4}+\frac{B}{x-2} \Rightarrow 3kx-18 \equiv A(x-2)+B(x+4)$ **or** $\frac{3kx-18}{(x+4)(x-2)} \equiv \frac{A}{x-2}+\frac{B}{x+4} \Rightarrow 3kx-18 \equiv A(x+4)+B(x-2)$	B1	1.1b
	$6k-18=6B \Rightarrow B=...$ or $-12k-18=-6A \Rightarrow A=...$ **or** $3kx-18 \equiv (A+B)x+4B-2A \Rightarrow A+B=3k,\ -18=4B-2A$ $\Rightarrow A=...$ or $B=...$	M1	1.1b
	$\frac{2k+3}{x+4}+\frac{k-3}{x-2}$	A1	1.1b
		(3)	
(b)	$\int \left(\frac{"2k+3"}{x+4}+\frac{"k-3"}{x-2}\right) dx = ...\ln(x+4)+...\ln(x-2)$	M1	1.2
	$("2k+3")\ln(x+4)+("k-3")\ln(x-2)$	A1ft	1.1b
	$("2k+3")\ln(5)-("k-3")\ln(5) \Rightarrow ("k+6")\ln 5 = 21 \Rightarrow k=...$	dM1	3.1a
	$(k=)\frac{21}{\ln 5}-6$	A1	2.2a
		(4)	
			(7 marks)

Notes

(a)

B1: Correct form for the partial fractions and sets up the correct corresponding identity which may be implied by two equations in A and B if they are comparing coefficients.

M1: Either

- substitutes $x=2$ or $x=-4$ in an attempt to find A or B in terms of k
- expands the rhs, collects terms and compares coefficients in an attempt to find A or B in terms of k

Or may be implied by one correct fraction (numerator **and** denominator)

You may see candidates substituting two other values of x and then solving simultaneous equations.

A1: Achieves $\frac{2k+3}{x+4}+\frac{k-3}{x-2}$ with no errors. Must be the correct partial fractions not just for correct numerators. May be seen in (b). Correct answer implies B1M1A1. One correct fraction only B0M1A0

(b)

M1: Attempts to find $\int \left(\dfrac{"2k+3"}{x+4} + \dfrac{"k-3"}{x-2} \right) dx$. Score for either $\dfrac{\dots}{x+4} \to \dots\ln(x+4)$ or $\dfrac{\dots}{x-2} \to \dots\ln(x-2)$

Allow the … to be in terms of k or just constants but there must be no x terms.

Condone invisible brackets for this mark.

A1ft: $("2k+3")\ln|x+4| + ("k-3")\ln|x-2|$

but condone round brackets e.g. $("2k+3")\ln(x+4) + ("k-3")\ln(x-2)$ or equivalent e.g. $("2k+3")\ln(x+4) + ("k-3")\ln(2-x)$

Follow through their partial fractions with numerators which must both be in terms of k.

Condone missing brackets as long as they are recovered later e.g. when applying limits.

dM1: A full attempt to find the value of k. To score this mark they must have attempted to integrate their partial fractions, substituted in the correct limits, subtracted either way round, set = 21 and attempted to solve to find k. Condone omission of the terms containing ln(1) or ln(-1).

Note that e.g. ln(-5) or ln(5) must be seen but may be disregarded **after** substitution and subtraction.

Do not be concerned with the processing as long as they proceed to $k = \dots$

Condone if they use x instead of k after limits have been used as long as the intention is clear.

A1: Deduces $(k=)\dfrac{21}{\ln 5} - 6$ or exact equivalent e.g. $\dfrac{21-6\ln 5}{\ln 5}$, $\dfrac{21-3\ln 25}{\ln 5}$.

Allow recovery from expressions that contain e.g. $\ln(-5)$ as long as it is dealt with subsequently.

Also allow recovery from invisible brackets. Condone $x = \dfrac{21}{\ln 5} - 6$

Some candidates may use substitution in part (b) e.g.

$$\int \left(\frac{"2k+3"}{x+4} + \frac{"k-3"}{x-2} \right) dx = \int \left(\frac{"2k+3"}{x+4} \right) dx + \int \left(\frac{"k-3"}{x-2} \right) dx$$

$$u = x+4 \Rightarrow \int \left(\frac{"2k+3"}{x+4} \right) dx = \int \left(\frac{"2k+3"}{u} \right) du = \dots\ln u$$

$$u = x-2 \Rightarrow \int \left(\frac{"k-3"}{x-2} \right) dx = \int \left(\frac{"k-3"}{u} \right) du = \dots\ln u$$

Score **M1** for integrating at least once to an appropriate form as in the main scheme e.g. …ln$u$

A1ft: For $("2k+3")\ln|u| + ("k-3")\ln|u|$

but condone $("2k+3")\ln u + ("k-3")\ln u$ which may be seen separately

Follow through their "A" and "B" in terms of k.

Condone missing brackets as long as they are recovered later e.g. when applying limits.

dM1: A full attempt to find the value of k. To score this mark they must have attempted to integrate their partial fractions using substitution, substituted in the correct changed limits and subtracts either way round, set = 21 and attempted to solve to find k. Do not be concerned with processing as long as they proceed to $k = \ldots$ Condone omission of terms which contain e.g. ln(1) or ln(-1).

Note that e.g. ln(-5) or ln(5) must be seen but may be disregarded **after** substitution and subtraction.

$$\left[(2k+3)\ln u\right]_1^5 + \left[(k-3)\ln u\right]_{-5}^{-1} = 21 \Rightarrow (2k+3)\ln 5 - (2k+3)\ln 1 + (k-3)\ln 1 - (k-3)\ln 5 = 21$$

$$\Rightarrow (2k+3)\ln 5 - (k-3)\ln 5 = 21 \Rightarrow (k+6)\ln 5 = 21 \Rightarrow k = \ldots$$

A1: $k = \dfrac{21}{\ln 5} - 6$ or exact equivalent e.g. $\dfrac{21 - 6\ln 5}{\ln 5}$, $\dfrac{21 - 3\ln 25}{\ln 5}$, $21\log_5 e - 6$.

Allow recovery from expressions that contain e.g. ln(−5) as long as it is dealt with subsequently.

Also allow recovery from invisible brackets.

Question	Scheme	Marks	AOs
6(a)	$3^{-2}\left(1+\frac{x}{3}\right)^{-2}=3^{-2}(1+...x+...x^2)$	M1	1.1b
	$(-2)\left(\frac{x}{3}\right)$ or $\frac{(-2)(-3)}{2!}\left(\frac{x}{3}\right)^2$	M1	1.1b
	$\left(1+\frac{x}{3}\right)^{-2}=1+(-2)\left(\frac{x}{3}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{3}\right)^2$	A1	1.1b
	$3^{-2}\left(1+\frac{x}{3}\right)^{-2}=\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}$	A1	2.1
		(4)	

(a)

M1: Attempts a binomial expansion by taking out a factor of 3^{-2} or $\frac{1}{3^2}$ or $\frac{1}{9}$ and achieves at least the first 3 terms in their expansion. May be seen separately e.g. $\frac{1}{9}$ and $(1+...x+...x^2)$

M1: A correct method to find either the x or the x^2 term unsimplified.

Award for $(-2)(kx)$ or $\frac{(-2)(-2-1)}{2!}(kx)^2$ where $k \neq 1$. Condone invisible brackets.

A1: For a correct unsimplified or simplified expansion of $\left(1+\frac{x}{3}\right)^{-2}$ e.g. $=1+(-2)\left(\frac{x}{3}\right)+\frac{(-2)(-3)}{2!}\left(\frac{x}{3}\right)^2-...$ or $1-\frac{2x}{3}+\frac{x^2}{3}-...$ Do not condone missing brackets unless they are implied by subsequent work.

Condone $\left(-\frac{x}{3}\right)^2$ for $\left(\frac{x}{3}\right)^2$

Also allow this mark for 2 correct simplified terms from $\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}$ with both method marks scored.

A1: $\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}$ cao Allow terms to be listed. Ignore any extra terms. Isw once a correct **simplified** answer is seen.

<u>Direct expansion, if seen, should be marked as follows:</u>

$$\left((3+x)^{-2}=3^{-2}-2\times3^{-3}\times x+\frac{-2(-2-1)}{2!}\times3^{-4}\times x^2\right)$$

M1: For $(3+x)^{-2}=3^{-2}+3^{-3}\times\alpha x+3^{-4}\times\beta x^2$

M1: A correct method to find either the x or the x^2 term unsimplified.

Award for $(-2)\times3^{-3}x$ or $\frac{(-2)(-2-1)}{2!}\times3^{-4}x^2$. Condone invisible brackets.

A1: For a correct unsimplified or simplified expansion of $(3+x)^{-2}$ e.g. $3^{-2}-2\times3^{-3}\times x+\frac{-2(-2-1)}{2!}\times3^{-4}\times x^2$

Also award for at least 2 correct simplified terms from $\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}$ with both method marks scored.

A1: $\frac{1}{9}-\frac{2x}{27}+\frac{x^2}{27}$ cao Allow terms to be listed. Ignore any extra terms. Isw once a correct **simplified** answer is seen.

Note that M0M1A1A0 is a possible mark trait in either method

Note regarding a possible misread in parts (b) and (c)

Some candidates are misreading $\int \frac{6x}{(3+x)^2}\,dx$ in parts (b) and (c) as $\int \frac{6}{(3+x)^2}\,dx$

If parts (b) and (c) are consistently attempted with $\int \frac{6}{(3+x)^2}\,dx$ then we will allow the M marks in (b) **only**. **M1** for $x^n \to x^{n+1}$ applied to their expansion in part (a) or 6× (their expansion in part (a)) and **dM1** for substituting in 0.4 and 0.2 and subtracting either way round (may be implied).
No marks are available in part (c)

MARK PARTS (b) and (c) TOGETHER

(b)	$\int 6x \times "\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)"\,dx = \int \left(\frac{2x}{3} - \frac{4x^2}{9} + \frac{2x^3}{9}\right)dx = ...$	**M1**	1.1b
	$\int \left(\frac{2x}{3} - \frac{4x^2}{9} + \frac{2x^3}{9}\right)dx = \frac{x^2}{3} - \frac{4x^3}{27} + \frac{x^4}{18}$ oe	**A1**	1.1b
	$\left["\frac{x^2}{3} - \frac{4x^3}{27} + \frac{x^4}{18}"\right]_{0.2}^{0.4} = \left(\frac{(0.4)^2}{3} - \frac{4(0.4)^3}{27} + \frac{(0.4)^4}{18}\right) - \left(\frac{(0.2)^2}{3} - \frac{4(0.2)^3}{27} + \frac{(0.2)^4}{18}\right)$	**dM1**	3.1a
	$=$ awrt 0.03304 or $\frac{223}{6750}$	**A1**	1.1b
		(4)	

(b)
M1: Attempts to multiply their expansion from part (a) by $6x$ or just x and attempts to integrate. Condone copying slips and slips in expanding. Look for $x^n \to x^{n+1}$ at least once having multiplied by $6x$ or x. Ignore e.g. spurious integral signs.
A1: Correct integration, simplified or unsimplified.

$$\frac{x^2}{3} - \frac{4x^3}{27} + \frac{x^4}{18} \text{ oe e.g. } \frac{1}{9}\left(3x^2 - \frac{4x^3}{3} + \frac{x^4}{2}\right),\ 6\left(\frac{x^2}{18} - \frac{2x^3}{81} + \frac{x^4}{108}\right)$$

If they have extra terms they can be ignored.
Ignore e.g. spurious integral signs.
dM1: An overall problem-solving mark for
- using part (a) by integrating $6x \times$ their binomial expansion and
- substituting in 0.4 and 0.2 and subtracting either way round (may be implied)

For evidence of using the correct limits we do not expect examiners to check so allow this mark if they obtain a value with (minimal) evidence of the use of the limits 0.4 and 0.2.
This could be e.g. $\left[\mathrm{f}(x)\right]_{0.2}^{0.4} = ...$ provided the first M was scored. If the integration was correct, evidence can be taken from answer of awrt 0.0330 if limits are not seen elsewhere. **Depends on the first M mark.**
A1: awrt 0.03304 (NB allow the exact value which is $\frac{223}{6750} = 0.033037037...$).
Isw following a correct answer.
Note answers which use additional terms in the expansion to give a different approximation score A0
Also note that the actual value is 0.032865…

Some may use integration by parts in (b) and the following scheme should be applied.

<u>Integration by parts in (b):</u>

Either by taking $u = 6x$ and $\frac{dv}{dx} = "\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)"$

$$\int 6x \times "\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)" dx = 6x \times \left(\frac{1}{9}x - \frac{x^2}{27} + \frac{x^3}{81}\right) - 6\int\left(\frac{1}{9}x - \frac{x^2}{27} + \frac{x^3}{81}\right)dx$$

$$= 6x\left(\frac{1}{9}x - \frac{x^2}{27} + \frac{x^3}{81}\right) - \left(\frac{1}{3}x^2 - \frac{2x^3}{27} + \frac{6x^4}{324}\right)$$

M1: A full attempt at integration by parts. This requires:

$$\int 6x \times "\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)" dx = kx \times f(x) - k\int f(x)dx = kx \times f(x) - kg(x)$$

Where f(x) is an attempt to integrate their expansion from (a) with $x^n \to x^{n+1}$ at least once **and** g(x) is an attempt to integrate their f(x) with $x^n \to x^{n+1}$ at least once

A1: Fully correct integration. Then **dM1A1** as in the main scheme

Or by taking $u = "\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)"$ and $\frac{dv}{dx} = 6x$

$$\int 6x \times "\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)" dx = 3x^2 \times \left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right) - \int 3x^2 \times \left(-\frac{2}{27} + \frac{2x}{27}\right)dx$$

$$= 3x^2 \times \left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right) - \int\left(\frac{6x^3}{27} - \frac{6x^2}{27}\right)dx = 3x^2 \times \left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right) - \left(\frac{6x^4}{108} - \frac{6x^3}{81}\right)$$

M1: A full attempt at integration by parts. This requires:

$$\int 6x \times "\left(\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}\right)" dx = kx^2 \times f(x) - k\int x^2 g(x)dx = kx^2 \times f(x) - kh(x)$$

Where f(x) is their expansion from (a) and g(x) is an attempt to differentiate their f(x) with $x^n \to x^{n-1}$ at least once **and** h(x) is an attempt to integrate their $x^2 g(x)$ with $x^n \to x^{n+1}$ at least once

A1: Fully correct integration. Then **dM1A1** as in the main scheme

(c)	Overall problem-solving mark (see notes)	**M1**	3.1a
	$u=3+x\Rightarrow\int_{3.2}^{3.4}\text{f}(u)\text{ d}u\Rightarrow\int_{3.2}^{3.4}\frac{6(u-3)}{u^2}\text{ d}u=\int_{3.2}^{3.4}\frac{6}{u}-\frac{18}{u^2}\text{d}u\Rightarrow...\ln u+...u^{-1}$	**M1**	1.1b
	$\int_{3.2}^{3.4}\frac{6(u-3)}{u^2}\text{ d}u=\int_{3.2}^{3.4}\frac{6}{u}-\frac{18}{u^2}\text{d}u\Rightarrow 6\ln u+18u^{-1}$	**A1**	1.1b
	$\left[6\ln u+18u^{-1}\right]_{3.2}^{3.4}=\left(6\ln 3.4+\frac{18}{3.4}\right)-\left(6\ln 3.2+\frac{18}{3.2}\right)=...$	**ddM1**	1.1b
	$6\ln\left(\frac{17}{16}\right)-\frac{45}{136}$ oe	**A1**	2.1
		(5)	
(c) Alt 1	Overall problem-solving mark (see notes)	**M1**	3.1a
	$\int 6x(3+x)^{-2}\text{ d}x=\frac{...x}{3+x}\pm...\int(3+x)^{-1}\text{d}x=\frac{...x}{3+x}\pm...\ln(3+x)$ oe	**M1**	1.1b
	$=6\ln(3+x)-\frac{6x}{3+x}$ oe	**A1**	1.1b
	$\left(6\ln(3+0.4)-\frac{6(0.4)}{3+0.4}\right)-\left(6\ln(3+0.2)-\frac{6(0.2)}{3+0.2}\right)=...$	**ddM1**	1.1b
	$6\ln\left(\frac{17}{16}\right)-\frac{45}{136}$ oe	**A1**	2.1

(c) Alt 2	Overall problem-solving mark (see notes)	**M1**	3.1a
	$\int 6x(3+x)^{-2}\text{ d}x=\int\left(\frac{...}{(3+x)}+\frac{...}{(3+x)^2}\right)\text{d}x=...\ln(3+x)+\frac{...}{3+x}$ oe	**M1**	1.1b
	$=6\ln(3+x)+\frac{18}{3+x}$ oe	**A1**	1.1b
	$\left(6\ln(3+0.4)+\frac{18}{3+0.4}\right)-\left(6\ln(3+0.2)+\frac{18}{3+0.2}\right)=...$	**ddM1**	1.1b
	$6\ln\left(\frac{17}{16}\right)-\frac{45}{136}$ oe	**A1**	2.1
			(13 marks)

Notes

(c) There are various methods which can be used

M1: An overall problem-solving mark for **all of**

- using an appropriate integration technique e.g. substitution, by parts or partial fractions – note that this may not be correct but mark positively if they have tried one of these approaches
- integrates one of their terms to a natural logarithm, e.g., $\frac{a}{3+x}\rightarrow b\ln(3+x)$ or $\frac{a}{u}\rightarrow b\ln u$
- substitutes in correct limits and subtracts either way round

M1: Integrates to achieve an expression of the required form for their chosen method

- substitution: $u=x+3\rightarrow\pm\frac{a}{u}\pm b\ln u$ or e.g. $u=(x+3)^2\rightarrow\pm\frac{a}{\sqrt{u}}\pm b\ln u$
- parts: $\pm a\ln(3+x)\pm\frac{bx}{3+x}$ condone missing brackets e.g. $...\ln x+3$ for $...\ln(3+x)$
- partial fractions: $\pm a\ln(3+x)\pm\frac{b}{3+x}$ condone missing brackets e.g. $...\ln 3+x$ for $...\ln(3+x)$

A1: Correct integration for their method e.g.

- substitution: $u = x+3 \rightarrow 6\ln u + 18u^{-1}$ or e.g. $u = (x+3)^2 \rightarrow 3\ln u + \frac{18}{\sqrt{u}}$
- parts: $6\ln(3+x) - \frac{6x}{3+x}$
- partial fractions: $6\ln(3+x) + \frac{18}{3+x}$ oe e.g. $3\ln(9+6x+x^2) + \frac{18}{3+x}$

Note that the above terms may appear "separated" but must be correct with the correct signs.
(ignore any reference to a constant of integration)
Do not condone missing brackets e.g. $6\ln x + 3$ for $6\ln(3+x)$ unless they are implied by later work.

ddM1: Substitutes in the correct limits for their integral and subtracts either way round to find a value
Depends on both previous method marks.
For evidence of using the correct limits we do not expect examiners to check so allow this mark if they obtain a value with (minimal) evidence of the use of the appropriate limits.
This could be e.g. $\left[f(x)\right]_{0.2}^{0.4} = ...$ provided both previous M marks were scored.
Note that for substitution they may revert back to 3 + *x* and so should be using 0.4 and 0.2

A1: A full and rigorous argument leading to $6\ln\left(\frac{17}{16}\right) - \frac{45}{136}$ or exact equivalent e.g. $3\ln\left(\frac{289}{256}\right) - \frac{45}{136}$ or e.g. $-6\ln\left(\frac{16}{17}\right) - \frac{45}{136}$

The brackets are not required around the $\frac{17}{16}$ and allow exact equivalents e.g. allow 1.0625 or $1\frac{1}{16}$ but not e.g. $\frac{3.4}{3.2}$. The $\frac{45}{136}$ must be exact or an exact equivalent. Also allow e.g. $6\ln\left|\frac{17}{16}\right| - \frac{45}{136}$

Ignore spurious integral signs that may appear as part of their solution.